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Generalization of the KKW Analysis for Black Hole Radiation

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Abstract

An extension of the Keski-Vakuri, Kraus and Wilczek (KKW) analysis to black hole spacetimes which are not Schwarzschild-type is presented. Preserving the regularity at the horizon and stationarity of the metric in order to deal with the across-horizon physics, a more general coordinate transformation is introduced. In this analysis the Hawking radiation is viewed as a tunnelling process which emanates from the non-Schwarzschild-type black hole. Expressions for the temperature and entropy of these non-Schwarzschild-type black holes are extracted. As a paradigm, in the context of this generalization, we consider the Garfinkle-Horowitz-Strominger (GHS) black hole as a dynamical background and we derive the modified temperature and entropy of GHS black hole. Deviations are eliminated and corresponding standard results are recovered to the lowest order in the emitted shell of energy. The extremal GHS black hole is found to be non-"frozen" since it is characterized by a constant non-zero temperature. Furthermore, the modified extremality condition forbids naked singularities to form from the collapse of the GHS black hole.

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Introduction

The idea of Keski-Vakkuri, Kraus and Wilczek [1, 2, 3] (KKW) has been applied till now to Schwarzschild-type black hole geometries. In this semiclassical analysis the energy conservation plays a dominant role since it leads us to a dynamical Schwarzschild-type black hole background which in turn leads to a more realistic description of the black hole radiance [4, 5]. The total Arnowitt-Deser-Misner mass [6] is fixed while the mass of the Schwarzschild-type black hole decreases due to the emitted radiation. A coordinate transformation is implemented so that the line element be non-singular at the horizon. This permits the study of across-horizon physics such as the black hole radiation. Using this methodology, it is has been made possible to exactly evaluate the non-thermal spectrum of the black hole radiation (the non-thermal character of the black hole radiation was already known since the seminal work of Hawking in mid-seventies). A direct byproduct of this analysis is that the black hole temperature is not only a function of the characteristics of the black hole but also of the energy of the emitted shell of energy. Additionally the black hole entropy is not that given by the area formula of Bekenstein and Hawking for the corresponding Schwarzschild-type black hole.

In the seminal works of Keski-Vakkuri, Kraus and Wilczek [1, 2, 3], although the starting point was spherically symmetric geometries, the analysis was restricted to Schwarzschild-type black holes, i.e. no general expressions for temperature and entropy were extracted for the above-mentioned geometries. Here we introduce a more general coordinate transformation in order to apply the KKW analysis to non-Schwarzschild-type black hole spacetimes. This general transformation preserves two conditions: (a) the regularity at the horizon which ensures that we are able to study across-horizon physics, (b) the stationarity of the non-static metric which implies that the time direction is a Killing vector, which are crucial in order to generalize the KKW analysis. The methodology followed is an appropriate modification of that adapted in the Schwarzschild-type black holes. The effect of this generalization in our calculation leads to exact expressions for the temperature and the entropy of the non-Schwarzschild-type black holes which are not anymore the Hawking temperature and the Bekenstein-Hawking entropy (given by the area formula), respectively.

The outline of this paper is as follows. Section 1 is devoted to the presentation of KKW analysis in Schwarzschild-type black hole geometries. In Section 2 we extend the analysis of Section 1 to the case of non-Schwarzschild-type black hole geometries. In the framework of this semiclassical analysis, we derive exact expressions for the temperature and entropy of these black hole spacetimes. In Section 3 we implement the above-mentioned expressions for the case of the GHS black hole. We derive the corresponding modified temperature and entropy of the GHS black hole and in the lowest order of the emitted shell of energy standard results, i.e. Hawking temperature and Bekenstein-Hawking entropy are reproduced, respectively, as a verification of the validity of our results. We consider the extremality condition which now will be shifted since the charge Q of the four-dimensional GHS black hole will be reached by the mass M earlier. The temperature of the extremal GHS black hole is shown to be non-zero and singularities are found to be always hidden behind the event horizon of the GHS black hole. Finally, in Section 4 we end up with a short summary and concluding remarks.

1 KKW Analysis

The idea of Keski-Vakkuri, Kraus and Wilczek (KKW) was firstly utilized for the case of the four-dimensional Schwarzschild black hole by Parikh and Wilczek [7]. From that time till now the KKW methodology was also applied to several black hole solutions such as (d+1)-dimensional Anti-de-Sitter black hole [8], AdS(2) black hole [9], two-dimensional charged black hole solution derived from the effective string theory at the low-energy limit [10], two-dimensional charged (and uncharged) dilatonic black holes [11] (dimensionally reduced from (2+1) spinning (and spinless) BTZ black holes), (2+1)-dimensional charged BTZ black hole [12, 13] and lately to a Schwarzschild-de Sitter spacetime [14].

All these black hole backgrounds belong to the same family of geometries since their line elements were Schwarzschild-like, i.e. they were of the type

$$ds^2 = -A(r)dt^2 + A^{-1}(r)dr^2 + r^2 d\Omega \tag{1}$$

where the metric function $A(r)$ had at least one (outer) event horizon (r_+), i.e.

$$A(r_+) = 0 \ . \tag{2}$$

Since the frame of the KKW methodology is the Hawking phenomenon which main contribution comes from the event horizon, the line element should be regular at the event horizon. Therefore a choice of suitable coordinates was enforced and the ansantz is to pick up the *a lá* Painlevé [15] coordinate transformation

$$\sqrt{A(r)}\ dt = \sqrt{A(r)}\ d\tau - \sqrt{\frac{1-A(r)}{A(r)}}\ dr \tag{3}$$

where τ is the new time coordinate (Painlevé coordinate).
After squaring expression (3) and substituting into equation (1), the line element becomes

$$ds^2 = -A(r)d\tau^2 + 2\sqrt{1-A(r)}d\tau dr + dr^2 + r^2 d\Omega \ . \tag{4}$$

It is obvious that there is no singularity at the event horizon r_H and these coordinates are stationary, but not static.
The radial null geodesics are given by

$$\dot{r} \equiv \frac{dr}{d\tau} = \pm 1 - \sqrt{1-A(r)} \tag{5}$$

where the upper (lower) sign in the above equation corresponds (under the assumption that τ increases towards future) to the outgoing (ingoing) geodesics.
At this point we will take into consideration the self-gravitation effect by fixing the total Arnowitt-Deser-Misner mass (M_{ADM}) of the black hole and letting the mass M of the black hole to vary. A shell of energy ω is now radiated by the black hole. It travels on the outgoing geodesics which now

are due to the fluctuation of the mass M of the black hole, derived by the modified line element

$$ds^2 = -A(r, M-\omega)d\tau^2 + 2\sqrt{1 - A(r, M-\omega)}d\tau dr + dr^2 + r^2 d\Omega \ . \qquad (6)$$

The outgoing radial null geodesics followed by the shell of energy will also be modified as

$$\dot{r} = 1 - \sqrt{1 - A(r, M-\omega)} \ . \qquad (7)$$

At this point let us remind ourselves of the following statements

1. It is known that the emission rate Γ for a radiating source [5] is given as

$$\Gamma \approx e^{-\beta\omega} = e^{+\Delta S_{bh}} \qquad (8)$$

   where β is the inverse temperature (T_{bh}) of the black hole and ΔS_{bh} is the change in the entropy of the black hole before and after the emission of the shell of energy ω (outgoing massless particle)

$$\Delta S_{bh} = S_{bh}(M-\omega) - S_{bh}(M) \ . \qquad (9)$$

2. A canonical Hamiltonian treatment gives a simple result for the total action of a system [1]

$$\mathcal{I} = \int d\tau \left[p_\tau + \frac{dr}{d\tau} p_r \right] \qquad (10)$$

   where τ and r are the Painlevé coordinate while p_τ and p_r are the corresponding conjugate momenta.

3. A semiclassical (WKB) approximation gives the following expression for the emission rate [3]

$$\Gamma \approx e^{-2Im\mathcal{I}} \qquad (11)$$

   where only the second term in equation (10) contributes to the imaginary part of the action.

We will consider here only the s-wave emission of massless particles. Therefore, using the above mentioned statements to the KKW methodology the imaginary part of the action can be obtained

$$Im\mathcal{I} = Im\int \frac{dr}{d\tau}\, p_r d\tau \tag{12}$$

$$= Im\int_{r_+(M)}^{r_+(M-\omega)} p_r dr \tag{13}$$

where r_+ is the outer event horizon of the black hole as mentioned before. It is useful to apply the Hamilton's equation

$$\dot{r} = \frac{dH}{dp_r} \tag{14}$$

$$= \frac{d(M-\omega)}{dp_r} \tag{15}$$

and thus

$$dp_r = \frac{d(M-\omega)}{\dot{r}} \tag{16}$$

Equation (13) can be written as

$$Im\mathcal{I} = Im\int_{r_+(M)}^{r_+(M-\omega)} \int_0^{p_r} dp_r' dr \tag{17}$$

and substituting equation (16) we get

$$Im\mathcal{I} = Im\int_{r_+(M)}^{r_+(M-\omega)} \int_0^{+\omega} \frac{d(M-\omega')}{\dot{r}} dr \ . \tag{18}$$

Thus if we use the modified outgoing geodesics (7) the imaginary part of the action will be written as

$$Im\mathcal{I} = Im\int_{r_+(M-\omega)}^{r_+(M)} \int_0^{+\omega} \frac{d\omega'}{1-\sqrt{1-A(r,M-\omega')}} dr \ . \tag{19}$$

It is easily seen, using the first statement, that the temperature and the entropy of the black hole is not the Hawking temperature (T_H) and the entropy given by the area formula of Bekenstein and Hawking (S_{BH}), respectively. Both of them are modified due to the specific modelling of the self-gravitation effect. Thus, the modified temperature and entropy of the black hole is given, respectively, by

$$T_{bh} = \frac{\omega}{2}\left\{Im\int_{r_+(M-\omega)}^{r_+(M)}\int_0^{+\omega}\frac{d\omega'}{1-\sqrt{1-A(r,M-\omega')}}dr\right\}^{-1} \tag{20}$$

$$S_{bh}(M-\omega) = S_{bh}(M) - 2Im\int_{r_+(M-\omega)}^{r_+(M)}\int_0^{+\omega}\frac{d\omega'}{1-\sqrt{1-A(r,M-\omega')}}dr \tag{21}$$

where the entropy of the black hole with mass M must be equal to that given by the area formula of Bekenstein and Hawking (S_{BH}). Therefore, the expression for the modified entropy will be

$$S_{bh}(M-\omega) = S_{BH} - 2Im\int_{r_+(M-\omega)}^{r_+(M)}\int_0^{+\omega}\frac{d\omega'}{1-\sqrt{1-A(r,M-\omega')}}dr\ . \tag{22}$$

2 Generalizing the KKW Analysis

The KKW methodology was, till now, restricted to Schwarzschild-type black hole backgrounds (see equation (1)). In this section we extend the KKW analysis to more general black hole backgrounds of the form

$$ds^2 = -A(r)dt^2 + B^{-1}(r)dr^2 + r^2 d\Omega \tag{23}$$

where $A(r)$ and $B(r)$ are functions satisfying the equation

$$A(r)\cdot B^{-1}(r) \neq 1\ . \tag{24}$$

The metric function $B(r)$ had at least one (outer) event horizon (r_+), i.e.

$$B(r_+) = 0 \tag{25}$$

since a event horizon appears at a spacetime point where $g^{rr} = 0$ and for the line element in (23)

$$g^{rr} = B(r)\ . \tag{26}$$

In order for the total Arnowitt-Deser-Misner mass (M_{ADM}) to be well-defined we restrict the class of metrics to those which are asymptotically flat, i.e.

$$A(r) \to 1 \quad \text{as} \quad r \to +\infty$$
$$B(r) \to 1 \quad \text{as} \quad r \to +\infty\ .$$

It is obvious that since we would like to deal with the radiation phenomenon of a black hole we need to keep the regularity at the event horizon and the stationarity of the metric which in turn implies that the time direction is a Killing vector [16, 17]. Therefore, we introduce the following *a lá* Painlev*é* more general, compared to that applied before for the Schwarzschild-type black holes, coordinate transformation

$$\sqrt{A(r)}\, dt = \sqrt{A(r)}\, d\tau - \sqrt{B^{-1}(r) - 1}\, dr \tag{27}$$

where τ is the new time coordinate (Painlev*é* coordinate). Substituting expression (27) in equation (23) the line element becomes

$$ds^2 = -A(r)d\tau^2 + 2\sqrt{\frac{A(r)}{B(r)}\Big(1 - B(r)\Big)}\, d\tau dr + dr^2 + r^2 d\Omega\ . \tag{28}$$

The radial null geodesics are now given by

$$\dot{r} = \sqrt{\frac{A(r)}{B(r)}}\left[\pm 1 - \sqrt{1 - B(r)}\ \right] \tag{29}$$

where the upper (lower) sign in the above equation corresponds, as before, to the outgoing (ingoing) geodesics under the assumption that τ increases towards future.

At this point we fix the total Arnowitt-Deser-Misner mass (M_{ADM}) of the black hole since we want to include the effect of self-gravitation. On the contrary, we let the mass M of the black hole to fluctuate. A shell of energy ω which constitutes of massless particles considering only the s-wave part of emission, is now radiated by the black hole. The massless particles travel on the outgoing geodesics which now are due to the varying mass M of the black hole, derived by the modified line element

$$ds^2 = -A(r, M-\omega)d\tau^2 + 2\sqrt{\frac{A(r, M-\omega)}{B(r, M-\omega)}\Big(1 - B(r, M-\omega)\Big)}d\tau dr + dr^2 + r^2 d\Omega\ . \tag{30}$$

The outgoing radial null geodesics followed by the massless particles, i.e. the shell of energy, will also be modified as follows

$$\dot{r} = \sqrt{\frac{A(r, M-\omega)}{B(r, M-\omega)}}\left[\pm 1 - \sqrt{1 - B(r, M-\omega)}\ \right]\ . \tag{31}$$

We adopt the previously mentioned three statements. We follow the same steps as before in order to write down an expression for the imaginary part of the action. Using the modified outgoing geodesics (31), the imaginary part of the action will now be given by

$$Im\mathcal{I} = Im\int_{r_+(M-\omega)}^{r_+(M)}\int_0^{+\omega}\frac{d\omega'}{\sqrt{\frac{\widetilde{A}}{\widetilde{B}}}\left[1-\sqrt{1-\widetilde{B}}\,\right]}dr \tag{32}$$

where $\widetilde{A}$ and $\widetilde{B}$ are defined as follows

$$\begin{aligned} \widetilde{A} &= A(r, M-\omega') &(33)\\ \widetilde{B} &= B(r, M-\omega')\ . &(34) \end{aligned}$$

Finally considering a more general black hole geometry than that of the Schwarzschild-type, the expression for the modified temperature is given as

$$T_{bh} = \frac{\omega}{2}\left\{Im\int_{r_+(M-\omega)}^{r_+(M)}\int_0^{+\omega}\frac{d\omega'}{\sqrt{\frac{\widetilde{A}}{\widetilde{B}}}\left[1-\sqrt{1-\widetilde{B}}\,\right]}dr\right\}^{-1} \tag{35}$$

and the corresponding expression for the modified entropy is given as

$$S_{bh}(M-\omega) = S_{BH} - 2Im\int_{r_+(M-\omega)}^{r_+(M)}\int_0^{+\omega}\frac{d\omega'}{\sqrt{\frac{\widetilde{A}}{\widetilde{B}}}\left[1-\sqrt{1-\widetilde{B}}\,\right]}dr\ . \tag{36}$$

It is obvious that in case where the metric functions A and B satisfy the condition

$$A(r)\cdot B^{-1}(r) = 1\,, \tag{37}$$

equations (35) and (36) coincide with the respective expressions (20) and (22) for the Schwarzschild-type black hole.

3 GHS Black Hole

The starting point will be the four-dimensional low-energy action obtained from string theory which is written in terms of the string metric as [18]

$$S = \int d^4x\sqrt{-g}\,e^{-2\phi}\left[-R-4\left(\nabla\phi\right)^2+F^2\right] \tag{38}$$

and the charged black hole metric is

$$ds^2_{\text{string}} = -\frac{\left(1-\frac{2Me^{\phi_0}}{r}\right)}{\left(1-\frac{Q^2e^{3\phi_0}}{Mr}\right)}dt^2 + \frac{dr^2}{\left(1-\frac{2Me^{\phi_0}}{r}\right)\left(1-\frac{Q^2e^{3\phi_0}}{Mr}\right)} + r^2 d\Omega \ . \qquad (39)$$

This metric describes a black hole with an event horizon at

$$r_+ = 2Me^{\phi_0} \qquad (40)$$

when $Q^2 < 2e^{-2\phi_0}M^2$. The Hawking temperature of the GHS black hole solution (39) easily evaluated by the use of the periodicity of the Euclidean section, is given by

$$T_H = \frac{1}{8\pi Me^{\phi_0}} \ . \qquad (41)$$

It is obvious that the Hawking temperature of the GHS black hole is independent of the charge Q, for $Q < \sqrt{2}e^{-\phi_0}M$.

At extremality, i.e. when $Q^2 = 2e^{-\phi_0}M^2$, the GHS black hole solution (39) becomes

$$ds^2_{\text{string}} = -dt^2 + \left(1-\frac{2Me^{\phi_0}}{r}\right)^{-2} dr^2 + r^2 d\Omega \ . \qquad (42)$$

and the corresponding Hawking temperature of the extremal GHS black hole (42) is

$$T_H^{\text{ext}} = 0 \qquad (43)$$

since the Euclidean section is smooth but without identifications.

In order to implement the methodology introduced in the previous section we firstly identify the metric functions $A(r)$ and $B(r)$ for the case of GHS black hole by comparing equation (23) with (39) and we get

$$A(r) = \frac{\left(1-\dfrac{2Me^{\phi_0}}{r}\right)}{\left(1-\dfrac{Q^2e^{3\phi_0}}{Mr}\right)} \qquad (44)$$

$$B(r) = \left(1-\frac{2Me^{\phi_0}}{r}\right)\left(1-\frac{Q^2e^{3\phi_0}}{Mr}\right) \ . \qquad (45)$$

Avoiding to make the complicated computation of the integral in expression (32) for the specific black hole background (39) we make the following approximation

$$\sqrt{\frac{\widetilde{A}}{\widetilde{B}}}\left(1-\sqrt{1-\widetilde{B}}\ \right) \approx \frac{1}{2}\sqrt{\widetilde{A}\widetilde{B}}\ . \tag{46}$$

Substituting expression (46) in equation (32) and using the expressions of $A(r)$ and $B(r)$ for the GHS black hole, i.e. equations (44) and (45), respectively, we get

$$\begin{aligned} Im\mathcal{I} &\approx 2\,Im\int_{r_+(M-\omega)}^{r_+(M)}\int_0^{+\omega}\frac{d\omega' dr}{\sqrt{\widetilde{A}\widetilde{B}}} \\ &= 2\,Im\int_{r_+(M-\omega')}^{r_+(M)}\int_0^{+\omega}\frac{d\omega' dr}{\left(1-\dfrac{2(M-\omega)e^{\phi_0}}{r}\right)}\ . \end{aligned} \tag{47}$$

We firstly perform the ω-integration which involves a contour integration into the lower half of ω' plane and we finally get

$$Im\mathcal{I} = \frac{\pi}{2}e^{-\phi_0}\left[r_+^2(M) - r_+^2(M-\omega)\right]\ . \tag{48}$$

Thus the modified temperature (35) for the case of the GHS black hole (39) is given as

$$T_{bh}(M,\phi_0,\omega) = \frac{\omega}{4\pi M^2 e^{\phi_0}}\left[1-\left(1-\frac{\omega}{M}\right)^2\right]^{-1} \tag{49}$$

and the corresponding expression for the modified entropy (36) is given as

$$S_{bh}(M,\phi_0,\omega) = S_{BH} - 4\pi M^2 e^{\phi_0}\left[1-\left(1-\frac{\omega}{M}\right)^2\right]\ . \tag{50}$$

We see that there are deviations from the standard results derived for a fixed background. The temperature of the GHS black hole is not the Hawking temperature (41) and its entropy is not given by the Bekenstein-Hawking area formula [19]

$$\begin{aligned} S_{BH} &= \frac{1}{4}\mathcal{A}_H \\ &= \pi r_+^2 = 4\pi M^2 e^{2\phi_0}\ . \end{aligned} \tag{51}$$

A welcomed but not unexpected result is that the modified temperature (49) evaluated to first order in ω yields the Hawking temperature of the GHS black hole (41). Additionally the modified entropy of the GHS black hole (50) to zeroth order in ω yields the corresponding Bekenstein-Hawking entropy (51). In the framework of our analysis, the extremal GHS black hole will be created when

$$Q^2 = 2e^{-\phi_0}(M - \omega)^2 \ . \tag{52}$$

It is obvious that the extremality condition ($r_+ = r_-$) is modified and the temperature of the extremal GHS black hole is no longer zero but it is given as

$$T_{bh}^{\text{ext}}(M, Q, \phi_0) = \frac{1}{4\pi M e^{-\phi_0}\left(1 - \dfrac{Q}{\sqrt{2}M}e^{-\phi_0}\right)} \ . \tag{53}$$

As a byproduct of this modification to the extremality condition, since the emitted shell of energy ω has to be always positive

$$\omega = M - \frac{Q}{\sqrt{2}}e^{\phi_0} > 0 \ , \tag{54}$$

it is implied that

$$Q < \sqrt{2}Me^{-\phi_0} \ . \tag{55}$$

Thus the extremality condition indicates that a naked singularity will never form from the collapse of the GHS black hole.

4 Conclusions

We have introduced a new, more general, coordinate transformation in order to generalize the KKW analysis to non-Schwarzschild-type black holes. Exact expressions for the temperature and the entropy of these black holes have been derived. Due to the specific modelling of the self-gravitation effect - described by the KKW analysis - the black hole temperature is not the Hawking temperature but depends explicitly on the emitted masssless (since we have restricted our analysis to the s-wave emission) particle's energy. The black hole entropy is also different from the corresponding entropy given by the area formula of Bekenstein and Hawking. It is easily seen that the modified entropy is less than the Bekenstein-Hawking entropy. In the context of our generalized KKW analysis, we have implemented the aforesaid expressions for the case of the static four-dimensional charged black holes in string theory which are the well-known Garfinkle-Horowitz-Strominger (GHS) black holes. The temperature of the GHS black hole is no more the corresponding Hawking temperature and the entropy of the GHS black hole is no longer the corresponding Bekenstein-Hawking entropy. The "greybody factors" showing up declare explicitly the dependence on the emitted particle's energy. As a verification of the validity of our generalized KKW analysis introduced here, the modified temperature and entropy of GHS black hole in the lowest order of the emitted shell of energy reproduce the standard results. Finally, we have shown that the extremal GHS black hole is no more "frozen" but it is characterized by a nonzero background temperature since the extremality condition is modified (due to the specific modelling of the backreaction effect). It should also be noted that because of the modified extremality condition naked singularities are forbidden in a natural way.

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